

# THERMAL FORWARD SCATTERING AMPLITUDES IN TEMPORAL GAUGES

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We employ the thermal forward scattering amplitudes technique in order to compute the gluon self-energy in a class of temporal gauges. The leading  $T^2$  and the sub-leading  $\ln(T)$  contributions are obtained for temperatures high compared with the external momentum. The logarithmic contributions have the same structure as the ultraviolet pole terms which occur at zero temperature (we have recently extended this result to the Coulomb gauge). We also show that the prescription poles, characteristic of temporal gauges, do not modify the leading and sub-leading high-temperature behavior. The one-loop calculation shows that the thermal self-energy *is transverse*. This result has also been extended to higher orders, using the BRS identities

There have been many investigations of thermal gauge field theories in the temporal gauge, both in the imaginary and in the real time formalisms<sup>1,2,3,4</sup>. One of the main advantages of the *non-covariant* temporal gauge is that it is physical and effectively ghost-free. At finite temperature, it may be considered a more natural choice, since the Lorentz invariance is already broken by the presence of the heat bath. It is also convenient for calculating the response of the QCD plasma to a chromo-electric field<sup>1,5</sup>. Despite these advantages, explicit calculations are known to be more involved than in covariant gauges, mainly because of the extra poles at  $q \cdot n = 0$  in the propagator, where  $q$  is the loop momentum and  $n = (n_0, \vec{0})$  ( $n_0^2 > 0$ ) is the temporal axial four-vector.

The standard method of calculation in the imaginary time formalism, employs the *contour integral formula*<sup>5</sup>

$$T \sum_{n=-\infty}^{\infty} f(q_0 = i\omega_n^\sigma) = \frac{1}{2\pi i} \oint_C dq_0 f(q_0) \frac{1}{2} \left[ \coth \left( \frac{1}{2} \beta q_0 \right) \right]^{i(2\sigma)}, \quad (1)$$

where

$$\omega_n^\sigma = \pi T (2n + \sigma); \quad \begin{cases} \sigma = 0 & (Bosons) \\ \sigma = 1 & (Fermions) \end{cases}$$

and the contour  $C$  is formed by two anti-

parallel straight lines equidistant from the imaginary axis. Of course, this formula cannot be employed when the function  $f(q_0)$  has poles along the imaginary axis. This is the reason why one should use some prescription for the poles in the temporal gauge<sup>4</sup> gluon propagator

$$\frac{1}{q^2} \left\{ -i\delta^{ab} \left[ g_{\mu\nu} - \frac{1}{q \cdot u} (q_\mu u_\nu + q_\nu u_\mu) + \frac{q_\mu q_\nu}{(q \cdot u)^2} \left( \frac{\alpha}{n_0^2} q^2 + 1 \right) \right] \right\}, \quad (2)$$

where  $u = n/n_0$  is the heat bath four-velocity.

For general covariant gauges, it is possible to show that all the thermal Green functions can be expressed (after the Cauchy integration in the complex plane) in terms of forward scattering amplitudes of on-shell thermal particles<sup>6,7</sup>. The main purpose of this work is to extend the forward scattering method to a class of temporal gauges. As an illustration of this technique, we will compute the *full tensor structure* of the one-loop gluon self-energy. (Previous calculations have considered only the static limit of the component  $\Pi_{00}$  of the self-energy<sup>1,4</sup>). In this way, we will be able to investigate the properties of transversality and gauge invariance of the leading contributions proportional to  $T^2$ . We also verify that *the gauge dependent sub-*

leading logarithmic part shares with the previous calculations in general covariant gauges <sup>7</sup> the interesting property of having the same structure as the *ultraviolet pole contributions* which occur at zero temperature.

The details involved in the forward scattering technique are explained in the appendix A of reference <sup>8</sup>. An important condition in order to be able to apply this technique is that the gluon propagator should have only the mass-shell poles at  $q^2 = 0$ . Therefore, our first task when using the axial gauge gluon propagator is to separate the contributions which can potentially have poles at  $n \cdot q = 0$  from the normal mass-shell poles. The simplest contribution having poles only at  $n \cdot q = 0$  is the ghost loop diagram shown in Fig. 1(c). At finite temperature it is proportional to

$$\int d^3\vec{q} \sum_{q_0} \left[ \frac{t_{\mu\nu}}{n \cdot q n \cdot (q+k)} + q \leftrightarrow -q \right], \quad (3)$$

where  $t_{\mu\nu}$  is a momentum independent quantity and  $q_0 = 2\pi i n T$  ( $n = 0, \pm 1, \pm 2, \dots$ ). Using partial fractions the integrand in (3) can be written as

$$\frac{1}{n \cdot k} \left[ \frac{1}{n \cdot q} - \frac{1}{n \cdot (q+k)} \right]. \quad (4)$$

Performing a shift  $q \rightarrow q - k$  in the second term, we can easily see that the ghosts *effectively decouple*. This shows that there is a simple mechanism for *some* cancellations of the temporal gauge poles, which involves only simple algebraic manipulations, *before* the computation of the sum over the Matsubara frequencies  $q_0$ . As we will see next, there are other contributions from the diagrams in Figs. 1(a) and 1(b) sharing same property.

The separation of the temporal gauge poles can be accomplished in a more systematic and physical way, using the following well known tensor decomposition of the gluon self-energy

$$\Pi_{\mu\nu}^{ab} = \delta^{ab} (\Pi_T P_{\mu\nu}^T + \Pi_L P_{\mu\nu}^L + \Pi_C P_{\mu\nu}^C + \Pi_D P_{\mu\nu}^D), \quad (5)$$

where

$$\begin{aligned} P_{\mu\nu}^T &= g_{\mu\nu} - P_{\mu\nu}^L - P_{\mu\nu}^D, \\ P_{\mu\nu}^L &= \frac{(u \cdot k k_\mu - k^2 u_\mu)(u \cdot k k_\nu - k^2 u_\nu)}{-k^2 |\vec{k}|^2}, \\ P_{\mu\nu}^C &= \frac{2k \cdot u k_\mu k_\nu - k^2 (k_\mu u_\nu + k_\nu u_\mu)}{k^2 |\vec{k}|}, \\ P_{\mu\nu}^D &= \frac{k_\mu k_\nu}{k^2}, \end{aligned} \quad (6)$$

where  $k^\mu P_{\mu\nu}^{T,L} = 0$ ,  $k^i P_{i\nu}^T = 0$ ,  $k^i P_{i\nu}^L \neq 0$  ( $i = 1, 2, 3$ ) and  $k^\mu P_{\mu\nu}^{C,D} \neq 0$ .

The calculation of structures  $\Pi_A$ ,  $\Pi_B$ ,  $\Pi_C$  and  $\Pi_D$  and the following algebraic manipulations were performed with the help of computer algebra. After partial fraction decompositions [as in Eq. (4)] and shifts  $q \rightarrow q - k$ , the rather involved expressions for  $\Pi_C$  and  $\Pi_D$  simplify considerably and all the temporal gauge poles at  $n \cdot q = 0$  cancel. We then proceed using the contour integral formula [Eq. (1)] and show that  $\Pi_C = \Pi_D = 0$ . It is interesting to note that, from the general relation <sup>9</sup>

$$\Pi_D = \frac{\Pi_C^2}{k^2 - \Pi_L} \quad (7)$$

the vanishing of  $\Pi_C$  to one loop order implies, in fact, that  $\Pi_D$  should vanish up to the three-loops order. Using the Becchi-Rouet-Stora identities <sup>10</sup> we have extended this result to all orders <sup>8</sup>.

For the structures  $\Pi_A$  and  $\Pi_B$  the temporal gauge poles do not cancel at the integrand level and we have to employ a prescription in order to be able to use the Eq. (1). Using the procedure described in the appendix of reference <sup>4</sup> we were able to show, by explicit calculation, that the *prescription poles* do not contribute to the leading and the subleading high temperature limit of  $\Pi_A$  and  $\Pi_B$ . Therefore, the thermal gluon self-energy can be represented, in the limit of high temperatures, in terms of *forward scattering amplitudes* of on-shell thermal gluons, as given by Eq. (6) of reference <sup>8</sup>.

In conclusion, our results show that the full tensor structure of the thermal gluon self-

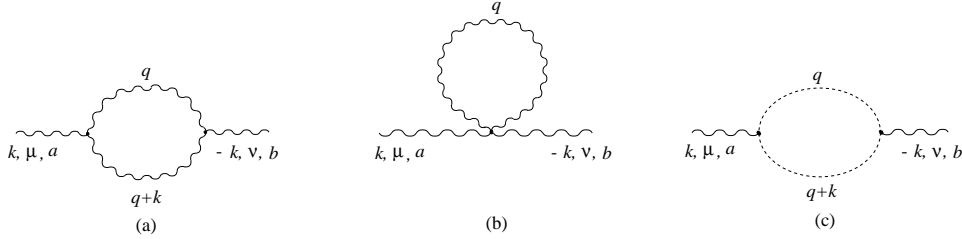


Figure 1. One-loop diagrams which contribute to the gluon-self energy. Wavy and dashed lines denotes respectively gluons and ghosts. All external momenta are inwards.

energy can be consistently computed in a class of temporal gauges, and expressed in terms of forward scattering amplitudes of on-shell thermal gluons. Using this approach, we have obtained the known gauge invariant result for the leading high temperature  $T^2$  contribution. We also have shown, by explicit calculation, that the one-loop thermal self-energy is *exactly transverse* for any temperature regime. Motivated by this result, we were able to prove the transversality to all orders. This property seems to be very peculiar to the temporal gauges. It is not valid, for instance, in general covariant gauges, except for the Feynman gauge, where the transversality has been verified only to one-loop order <sup>7</sup>.

Our approach also gives sub-leading contributions which are in agreement with the conjecture proposed in <sup>11</sup>, according to which the ultraviolet divergent contributions which arises at  $T = 0$  are identical to the thermal contributions proportional to  $\ln(1/T)$ . We have recently verified this conjecture also in the *Coulomb gauge* so that the divergent part of the Coulomb gauge gluon self-energy <sup>12,13</sup> can be alternatively obtained from our  $\ln(1/T)$  contribution. The details of this analysis will be reported elsewhere.

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